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ON FORECASTING CEILING LOWERING DURING CONTINUOUS RAIN

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ABSTRACT

During steady rain, the ceiling lowers in a discontinuous fashion. The ceiling heights may be predicted with sufficient accuracy by using a set of empirically determined rules. To obtain a relation for the time of occurrence of these ceilings, the factors which influence cloud formation are considered. An expression is derived for the rate of moisture increase due to evaporation from falling raindrops. The rate of moisture change, given by this expression, is combined with the effect of the other factors in order to obtain a formula which may be applied to find the time a ceiling of given height will occur. The variables in the forecast formula are (1) the wet-bulb temperature depression measured before the start of rain and (2) F_z, the effective rate of moisture increase caused by factors other than evaporation. Values for F_z are found empirically. An approximate method, based on the surface value of the depression, is used for finding the time of occurrence of the 800-, 500-, and 300-foot ceilings. This approximate method appears to be best suited for forecasting the 500-foot ceiling.

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INTRODUCTION

Low ceilings [1] associated with rain occurring in advance of warm fronts or well-developed cyclones frequently restrict flying over a wide area. At times the ceiling becomes low at the time rain starts, while at other times the weather may be flyable for several more hours. In either case low ceilings occur in a large area at about the same time, making it difficult for the pilot caught in this weather to find a suitable landing place.

Findeisen [1] considered the problem and derived a formula for the rate of downward growth of the fracto-cumuli forming underneath the rain cloud during steady precipitation. His derivation is based on the assumption that all rain forms by the melting of snow falling out of the nimbostratus, accompanied by cooling of the air just beneath the cloud. This cooling causes a steepening of the temperature gradient below the zero isothermal, consequently increasing convection which results in the formation of the fracto-cumuli. Assuming that the heat required to melt all the precipitation comes from the surrounding air, Findeisen shows that the rate of downward growth of the low cloud is given by:

$$\frac{dc}{dt}=6.2N \tag{1}$$

where N is the rate of rainfall in mm.hr.$^{-1}$ and dc/dt is the rate of lowering in cm.sec.$^{-1}$.

According to equation (1) clouds formed in rain lower at a rate directly proportional to the intensity of rainfall. An attempt to apply the result to forecast ceilings during

[1] The definition of the term "ceiling" is subject to change. In order that no confusion shall result, it will be employed here to mean the height, above the ground, of the base of the lowest cloud layer covering more than half the sky. This definition approximates the official meaning in effect at the time of the ceiling data used in this report. When no specification as to the amount of cloudiness is intended, terms such as "base height of cloud (or cloud layer)" or "height of base of cloud" will be employed.

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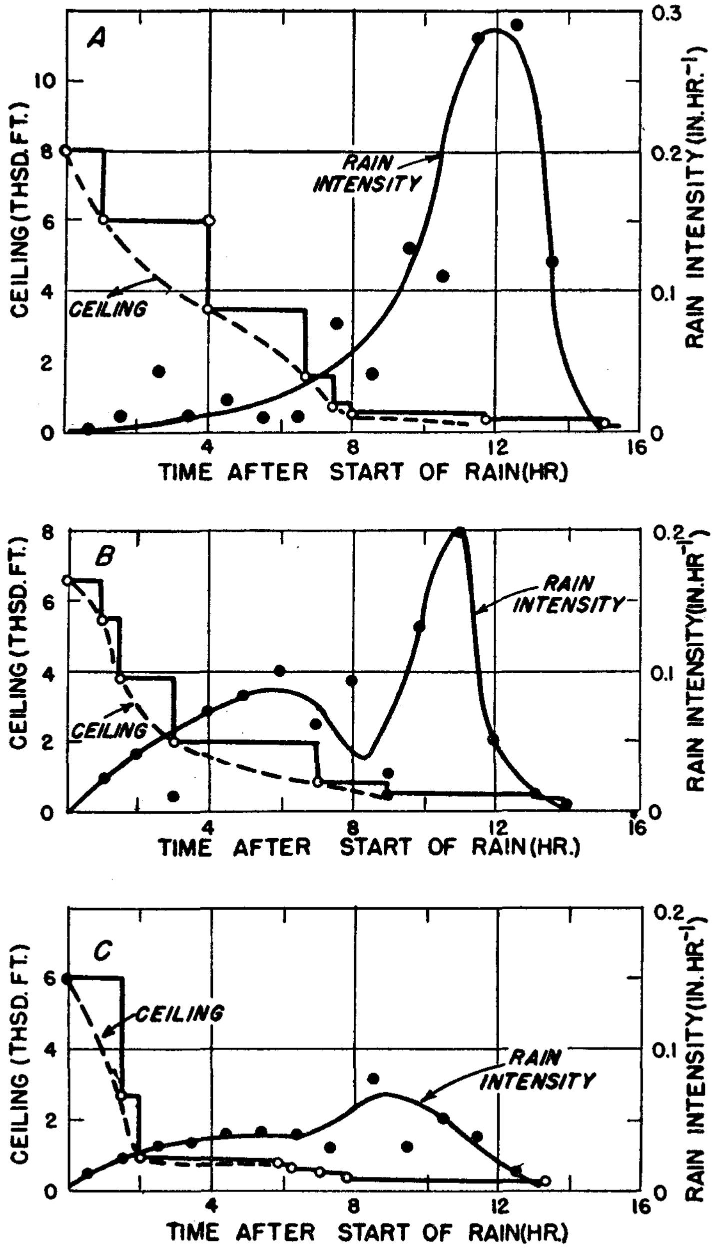

FIGURE 1.—Variation of ceiling and rain intensity in three typical cases observed at Boston. (A) April 24, 1944. (B) May 15, 1944. (C) March 7, 1948.

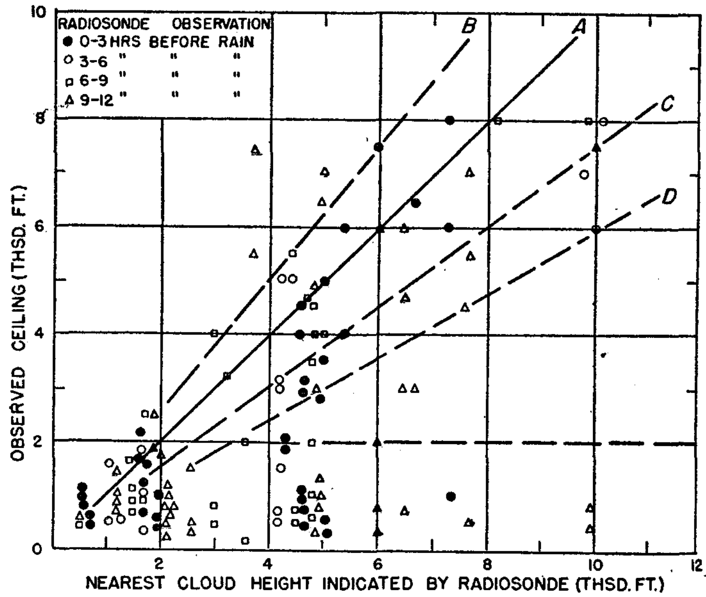

FIGURE 2.—Observed ceiling during rain plotted against the nearest level defined by rules (1) and (2) found from latest radiosonde preceding the rain. Portland, Maine, 1945–1947.

rain in eastern United States leads to failure, perhaps because conditions influencing cloud formation are somewhat different from those assumed by Findeisen.

Three typical cases of ceiling lowering during rain, observed at Boston, are shown in figure 1. The ceiling does not lower continuously, but appears to remain fixed until a cloud forms below this height and increases in amount sufficiently for its base height to become the ceiling. This ceiling then remains practically constant until another cloud layer, closer to the ground, appears and increases in amount so as to constitute the ceiling. In this manner the ceiling lowers discontinuously until a final cloud layer appears close to the ground. This final cloud layer may increase in amount and extend downward very slowly until the damp air is displaced by drier air.

The smoothed curves through the points representing ceiling and the corresponding time it was first observed are typical of those obtained for all cases and may be taken to define the variation of ceiling during continuous rain. Comparing the ceiling and rain intensity, it is seen that the ceiling lowers at a decreasing rate as the rain intensity increases, and that the minimum ceiling is reached before maximum rain intensity. Apparently the rate of ceiling lowering and rain intensity are negatively correlated.

With these observational facts in mind, it is the purpose of this study to examine rules which may be useful in forecasting variations of ceiling during rain, and, through a logical approach, to develop an objective method for predicting the rate of ceiling lowering during rain.

BASE HEIGHT OF CLOUD LAYERS DEVELOPING IN RAIN

Two generally well known rules are applied by the aviation forecaster to determine the base height of cloud layers likely to occur during rain. These are:

(1) The base of a cloud layer will be at a height where the temperature lapse rate changes from positive to less positive or negative.

(2) The base of a cloud layer will be at a height where the wet-bulb temperature depression, or dew point temperature depression, is a minimum.

To test these rules the Portland, Maine, surface reports for the years 1945, 1946, and 1947 were examined for all cases of continuous rain. All the differing ceilings reported during each rain period and all the heights defined by rules (1) and (2) found from the latest radiosonde preceding the rain were listed. In a few cases where multiple heights were indicated within a short distance, the lowest height was selected. Due to an incomplete file of radiosonde data available, several cases were omitted, leaving a total of 32 cases for study. All the ceilings reported in these cases are shown plotted against the nearest height given by the radiosonde in figure 2. If the two rules gave both the necessary and sufficient conditions for determining the height of cloud bases, all the points in figure 2 would fall near the straight line OA. However, all the points do not fall near OA, and if the rules are still considered sufficient, then the deviations must be due to (1) errors in measuring or estimating the ceiling, (2) time lag from the radiosonde observation to the time of ceiling observation, or (3) continuous lowering of ceiling. The points above OA, of course, can not be in error because of continuous lowering. The greatest deviations occur in the points above OB, determined from radiosonde data preceding the rain by more than 9 hours; it is likely that a stratum of higher temperature or humidity appeared at these levels in the meantime. The smaller deviations included between OA and OB relate to cases with little or no lag in observation time and can be attributed to the unavoidable error of estimating or measuring the ceiling. Since the error of estimate is as likely to be positive as to be negative, all the points included between lines OB and OC may be considered to agree with the rules for forecasting the height of cloud bases and, in particular, the ceiling during steady rain.

A relatively large number of points fall near or below 1,000 feet. Since there is a large separation between these points and the clouds above them, it is unlikely that the points correspond to ceilings which lowered continuously. Furthermore, the error in the measurement of the base height of low clouds is generally negligible. These ceilings are reported regardless of the time lag between radiosonde observation and start of rain, so it must be assumed that a temperature inversion develops near the surface after the start of rain, if not sooner. The wind almost always increases with the approach of continuous rain. Mechanical turbulence associated with the increasing wind results in a temperature inversion not far from the ground. A maximum relative humidity at the base of the mechanically produced inversion and the inversion itself may not be evident in a radiosonde taken before the start of rain. Strong winds result in higher inversions and a more rapid development of the inversion so that the number of ceilings between 1,000 and 2,000 feet not associated with an inversion is smaller than the number below 1,000 feet.

If the few cases lying between line OD and the 2,000-foot level are attributed to a lag in observations, then the ceilings falling between OC and OD may be due to downward growth of a cloud base. However, most of these points are close to, or above, OC, so considering the possible error in observing ceilings, the continuous lowering of ceiling during rain is negligible in most cases.

This leads to the following additional rules which may be found useful in forecasting the variation of ceiling during rain:

(3) If the rain is of sufficient duration, a ceiling will occur below 2,000 feet. Most frequently it is a ceiling of 800 feet.

(4) During continuous rain a ceiling generally does not occur at the height of temperature discontinuity and/or maximum humidity until after the occurrence of a ceiling corresponding to the next higher level of temperature discontinuity and/or maximum humidity.

(5) The ceiling remains practically constant until the next lower cloud layer appears and increases sufficiently for its base height to become the ceiling.

Applying the above rules to available radiosonde data leads to a reasonably accurate forecast of the ceilings which will occur during continuous rain. From the study of ceiling variation it is obvious that if the radiosonde observation is taken over 6 hours before the start of rain then a significant ceiling may occur which is not given by the rules. However, whether or not such a ceiling can occur may be determined readily by inspection of radiosonde data closer to the rain area, or if these are not available, by noting the base heights of clouds reported in the rain area.

Since the ceilings which occur, when the rain is of sufficient duration, can be found with the degree of accuracy required in an aviation forecast, it remains to develop a method of forecasting the time these ceilings will first occur.

FACTORS INFLUENCING TIME OF CLOUD FORMATION

From the manner in which the ceiling varies during continuous rain, it appears that the important factors influencing the variation may be:

(1) *Advection of warmer and more humid air at selected levels.* In these strata of warm humid air which appear in advance of the rain area, the relative humidity increases upstream, reaching the 100 percent value at the forward edge of the cloud sheet. The cloud itself may not move at the speed of the wind because other factors associated with the rain tend to increase the relative humidity of the air in the strata.

(2) *Vertical mixing.* Mechanical turbulence at the boundary between the warm stratum and the colder air beneath it and in the layer next to the ground causes vertical mixing which tends to increase the moisture content of the upper part of the mixed layer at the expense of the lower part. If the moisture content of the mixed layer is sufficiently high, a cloud will form near the top of the mixed layer, as shown by Petterssen [2]. The base height of this cloud will remain constant until the moisture content, due to other factors (e. g., evaporation and advection), increases sufficiently to lower the mixing condensation level (MCL). Thus, if successively lower layers

form in intervals of a few hours, the gradual lowering of the cloud basis, except the lowest, is negligible when the ceiling variation is considered.

(3) *Evaporation from falling raindrops.* Evaporation is effective in increasing the relative humidity of the entire air column. The rate of evaporation is greater when the dryness of the air is greater. Evaporation moistens the dry air between the moist strata rapidly, but evaporation diminishes as the relative humidity increases, and unless the rain is warmer than the wet-bulb temperature of the surrounding air, evaporation alone cannot produce condensation. When the air is very dry in the lower layer, evaporation determines the time of formation of the lowest clouds. Vertical mixing may produce an inversion near the ground, but if the air is dry, the MCL will lie above the layer of mixing and no cloud will form; however, evaporation will increase the amount of moisture rapidly—the drier the air, the more rapid the increase. Eventually the MCL falls within the layer of mixing and a cloud forms near the base of the turbulence inversion. This cloud builds downward as evaporation continues.

The problem of determining the time it would take for a cloud to form due to the combined effect of advection, vertical mixing, evaporation and, perhaps, other factors is a complex one. However the problem may be simplified by confining attention to evaporation and allowing for the other factors by inclusion of a suitable parameter. Since the effects of vertical mixing and advection are highly correlated, each depending on the wind and moisture distribution surrounding the rain area, a single parameter may suffice for the effects of the two.

MOISTURE INCREASE DUE TO EVAPORATION

If a falling raindrop is conceived to be surrounded by a thin viscous air film, through which heat is transferred by conduction, and this boundary layer to be surrounded by a turbulent zone, through which heat is transferred convectively, then a relation for the transfer of heat between the raindrop and the surrounding free air may be derived readily. Thus, Newton's Law, which is applicable to the boundary layer, may be written:

$$\frac{dQ}{dt}=h_f a(T_b-T_r) \qquad (2)$$

where dQ/dt is the rate of heat transfer through the film, a is the mean area of the film (which may be taken to be the surface area of the drop), T_r is the surface temperature of the raindrop and T_b is the temperature at the outer boundary of the air film. The heat conductance h_f is defined by the ratio k/δ, k being the conductivity of the film (which may be taken to be that for air) and δ the thickness of the film.

Taking equation (2) to define the "film" conductance of heat h_f, analogous equations are written to define the "convective" and "over-all" conductances, h_c and h_g, respectively, thus:

$$\frac{dQ}{dt}=h_c a(T-T_b) \qquad (3)$$

and

$$\frac{dQ}{dt}=h_g a(T-T_r) \qquad (4)$$

where T is the temperature of the free air.

If there is a continuous flow of heat between the raindrop and the free air, then $1/h_g=1/h_f+1/h_c$. By considering the dimensions of the variables upon which h_c and δ depend, i. e., the raindrop diameter d, its speed relative to the air v, the air viscosity μ, and the air density ρ, it can be shown that:

$$\frac{h_g d}{k}=\text{function of }\left(\frac{dv\rho}{\mu}\right)\equiv\phi(R)$$

where $R\equiv\frac{dv\rho}{\mu}$ is, by definition, the Reynolds Number for the raindrop. The form of the function ϕ may be found experimentally; however, except for a shape factor, which will be assumed to have the value of unity, the function may be approximated by the empirical relation:

$$\frac{h_g d}{k}=0.45+0.33(R)^{0.56} \qquad (5)$$

which is based on the correlation of data for the flow of air at right angles to the axes of single cylinders ranging in diameter from 0.001 to 0.375 inch [3]. Values of h_g for the range of sizes found in rain, computed by equation (5) and based on R values calculated by Gunn and Kinzer [4] and $k=0.0000568$ cal. cm.$^{-1}$ sec.$^{-1}$(°C)$^{-1}$ are listed in table 1. For sizes ranging from 0.05 to 0.50 cm. there is only a 12 percent variation of individual values from the mean value of 0.0042 c. g. s. units so it may be assumed that h_g is constant for all raindrops.

TABLE 1.—*Terminal velocity, Reynolds number and over-all heat conductance for raindrops of various size*

d *	v †	R ‡	h_g
Cm.	*Cm. sec.*$^{-1}$		*Cal. sec.*$^{-1}$ *cm.*$^{-2}$ *deg.*$^{-1}$
0.05	206	68.7	0.0046
.10	403	269	.0046
.15	541	542	.0045
.20	649	866	.0043
.25	742	1239	.0041
.30	806	1613	.0041
.35	852	1991	.0038
.40	883	2357	.0037
.45	900	2704	.0037
.50	909	3033	.0037

*Equivalent drop diameter calculated from the mass (Gunn and Kinzer [4]).
†Terminal velocity of fall for distilled water droplets in stagnant air at a pressure of 760 mm., temperature 20° C. and relative humidity of 50 percent (Gunn and Kinzer [4]).
‡Reynolds number = (air density) × (equivalent diameter) × (measured velocity) ÷ (viscosity of air). (Gunn and Kinzer [4].)

Now, if it is supposed that the heat required to evaporate rain comes from the surrounding air and that a state of equilibrium is reached instantaneously, then the temperature change dT, of a unit mass of air in the time interval dt, is

$$dT = -\frac{h_g A}{c_p}(T - T_w)dt \tag{6}$$

where T_w is the wet-bulb temperature, c_p the specific heat for air at constant pressure, and A the total raindrop area in unit mass of air. Neglecting variations in c_p, it remains to find the variation in A, in order to solve equation (6).

Lenard [5] measured the drop distributions in various types of rain and expressed the measurements in terms of the number of drops of each size falling on a unit horizontal area in a unit time. Using these data, the value of A near the ground may be computed from:

$$A = \frac{\pi}{\rho}\Sigma\left(\frac{n_i d_i^2}{v_i}\right) \tag{7}$$

where n_i is the number of drops of size d_i and terminal velocity v_i falling on a unit horizontal area in unit time. Values of A computed from this equation, together with a description of the rain and its intensity, as given by Lenard, are shown in table 2.

TABLE 2.—*Character of rain and surface area of raindrops*

Intensity	A	Character of rain
Mm. min.$^{-1}$	*Cm.*^{2} *gm.*$^{-1}$	
(1) 0.09	0.0083	Very ordinary looking rain.
(2) .06	.0097	Do.
(3) .11	.0058	Breaks occurred during which the sun shone.
(4) .05	.0045	Beginning of a thundershower.
(5) .32	.0115	Sudden rain from a small cloud.
(6) .72	.0292	Violent rain, like a cloudburst, some hail.
(7) .57	.0220	Heaviest period, less heavy period, and period
(8) .34	.0230	of stopping of a continuous fall which at times
(9) .26	.0085	took the form of a cloudburst.

There is a good correlation between the rain intensity and A, however the intensity values given by Lenard appear to be computed from the size distribution, which may account for this good correlation. To find the true relation between rain intensity and A, it is necessary to have data for measured rain intensity. These data are not available, so the type of rain will be considered instead.

At the beginning of rain, when the air is relatively dry, as in cases (3) and (4), A has a value from 0.004 to 0.006 cm^{2}gm^{-1}. The values increase to about 0.008 or 0.010 after the rain becomes steady, as in cases (1) and (2), and then remains at that value until stopping, case (9); but during rain of cloudburst intensity A may reach as high as 0.03 cm^{2}gm^{-1}. Since the minimum ceiling is generally observed to occur before the rain has reached its maximum intensity, it seems reasonable to assume A has the constant average value 0.005.

Equation (6) may now be solved readily to give:

$$\frac{\tau}{\tau_0} \equiv \frac{(T - T_w)}{(T - T_w)_0} = e^{-wt} \tag{8}$$

where $w \equiv \frac{h_g A}{c_p}$

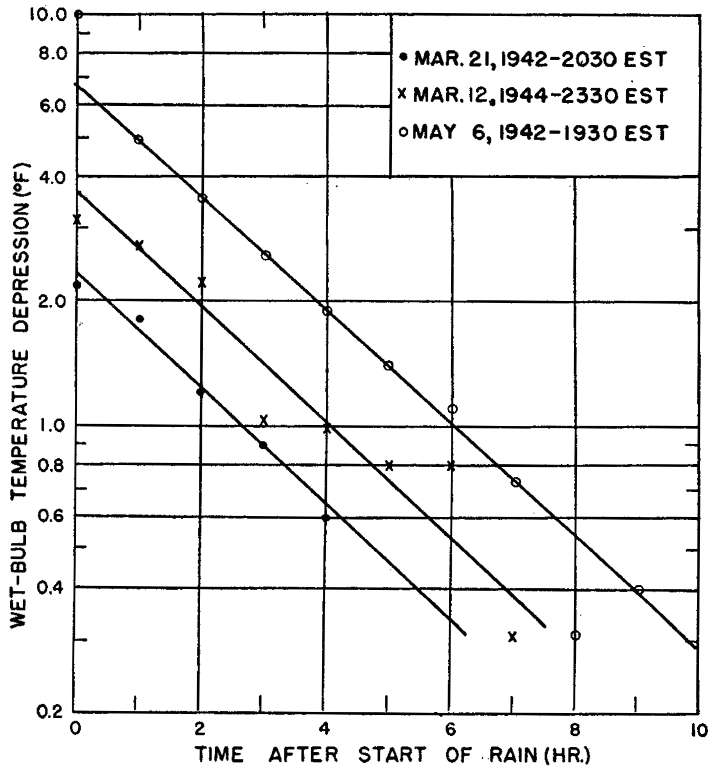

FIGURE 3.—Wet-bulb depression plotted against hours of rain for three cases of continuous rain at Boston, Mass.

and $\tau_0 \equiv (T - T_w)_0$ is the value of the wet-bulb depression measured at the start and $\tau \equiv (T - T_w)$ is the value at the end of the time interval t.

Using the values found for A and h_g, and $c_p = 0.24$ cal.gm.$^{-1}$ gives:

$$w = (0.0042)\ (0.005)\ (3600)/(0.24) = 0.3 \text{ hr.}^{-1}$$

If factors other than evaporation may be neglected, then w may be computed directly from equation (8). These factors may be assumed negligible when the air is dry at the start of rain and the wind is light during the rain. However surface variations of depression only are available and, since the diurnal variation at the surface may be appreciable, the diurnal factors would have to be considered. The normal diurnal variation of the wet-bulb depression shows an almost constant value during the night, so to determine w from the surface variation it is best to select cases in which rain began during late evening.

A case in which evaporation appears to be the factor controlling the wet-bulb temperature depression occurred at Boston on May 6, 1942. Rain began at 1930 EST. The wind was SSW 15 to 20 m. p. h. until 2 hours before the start of rain when it diminished to less than 10 m. p. h. and then remained gentle for the remainder of the night. The wet-bulb depression plotted against hours of rain on semilogarithmic paper (fig. 3) gives a straight line of slope 0.30 hr^{-1}, thus verifying equation (8) and the value for w found indirectly. Two other cases plotted in the

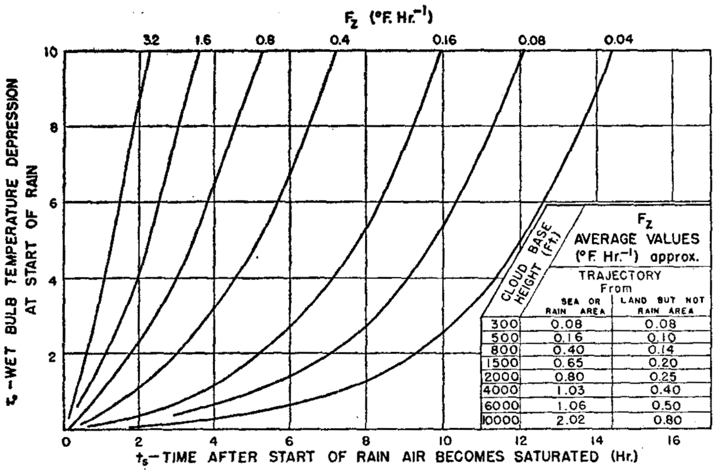

FIGURE 4.—Nomograph based on equation (10) giving t_s as a function of F_s and τ_0. The average values of F_s given in the table were determined from Portland, Maine, data using equation (12) or (13) (see fig. 5).

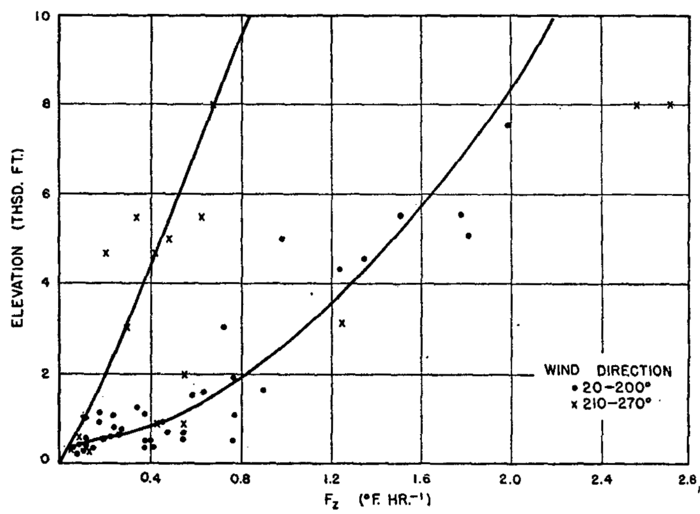

FIGURE 5.—Values of F_s by elevation determined from Portland, Maine, data using equation (12) or (13).

same manner can also be fitted with straight lines of the same slope. In these cases the wind was moderate to fresh, but steady, and, no doubt, advection and vertical mixing were appreciable.

TIME OF OCCURRENCE OF A CEILING OF GIVEN HEIGHT

If rain is at its equilibrium temperature then, according to equation (8), evaporation alone can not result in the formation of a cloud, since $t=\infty$ when $\tau=0$. However evaporation results in an exponential decrease in the depression, so it may well determine the time of cloud development, at least at times when the air is relatively dry at the start of rain.

Suppose factors, other than evaporation, cause an independent decrease in the wet-bulb temperature depression, say F_z per unit time, then the total rate of change of the depression during rain is

$$\frac{d\tau}{dt}=-(w\tau+F_z) \tag{9}$$

If it is assumed that F_z varies with height only, then the time t_s, after the start of rain, when the air at a given height becomes saturated is

$$t_s=\frac{1}{w}\log_e\left(1+\frac{w}{F_s}\tau_0\right) \tag{10}$$

where τ_0 is the value of the depression at that height at the time rain begins. Equation (10) is represented graphically in figure 4.

If the depression is measured t' hours before rain starts and has a value τ' at that time, then since F_z has been assumed constant, the depression at the start of rain is

$$\tau_0=\tau'-F_zt' \tag{11}$$

which may be subsitiuted in equation (10) to find t_s. However, if $\tau'<F_zt'$, then the time of saturation, t'_s, in hours after the time τ' is measured is simply:

$$t'_s=\frac{\tau'}{F_z} \tag{12}$$

Obviously use of equations (11) and (12) will underestimate the time of saturation; the error will be greatest when τ' is measured long before the start of rain, i. e., before F_z becomes constant. In most cases F_z can not be determined directly, so it will be considered sufficient for this study to obtain some empirical values suitable for forecasting the time a ceiling of given height will occur. Such values may be computed readily from a combination of equations (10) and (11), i.e.,

$$F_z=\frac{\tau'}{t'+\frac{1}{w}(e^{wt_s}-1)} \tag{13}$$

or from equation (12), when the time of saturation is close to the time rain begins. Now, if the time a ceiling of given height first occurs is taken for t_s (or t'_s) then the values for F_z determined by using equations (12) or (13) may be considered appropriate corrected values for finding the time a ceiling of given height will occur.

Such values of F_z were found from the Portland, Maine data and are shown plotted in figure 5. Evidently the points fall in two classes; in one, F_z is relatively large, and in the other it is small. From the winds aloft last reported near the start of rain, it was found that the smaller set is associated with winds having a direction of 210° to 270°, and the larger set of values associated with a wind direction of 20° to 200°—except for a few cases.

It appears that the higher values correspond to cases in which the air trajectory passes either over water or close to the center of the rain area; while the lower values correspond to cases in which the air has a land trajectory not passing close to the rain area center. For places on the North Atlantic Seaboard, such as Portland and Boston, when continuous rain is associated with secondary Lows traveling northward, the higher F_z values should generally be applicable. The smoothed values given in figure 4 are

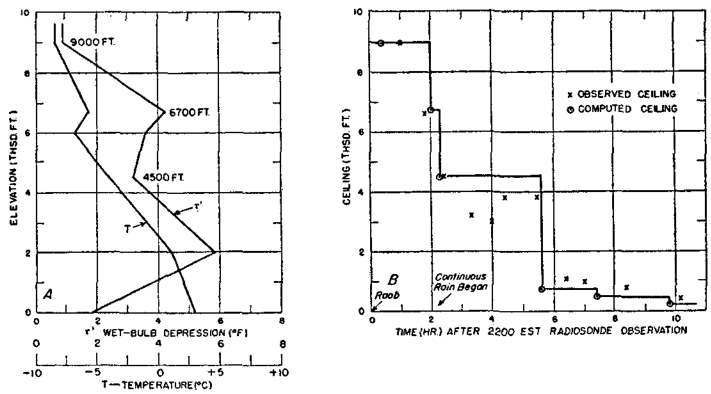

FIGURE 6.—(A) Temperature (T) and wet-bulb temperature depression (τ') from Portland, Maine, sounding, 2200 EST, April 10, 1946. (B) Comparison of computed with observed ceiling at Portland, Maine, April 10–11, 1946.

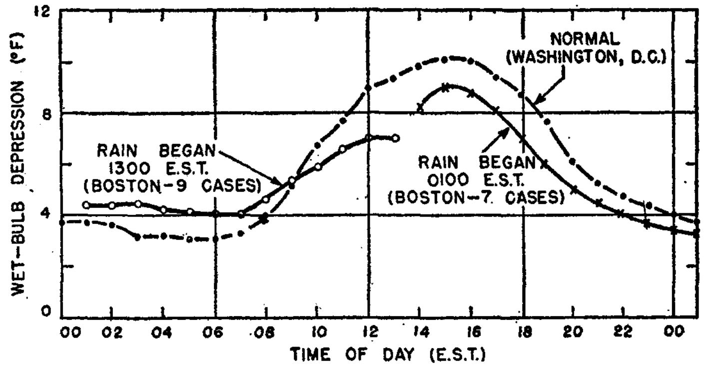

FIGURE 7.—Diurnal variation of wet-bulb temperature depression.

based on insufficient data to be considered very reliable, however they may be used until better estimates are made. To illustrate the application of the results obtained so far, we now consider an example forecast of ceiling during continuous rain based on upper air data.

EXAMPLE OF USE OF UPPER AIR DATA TO FORECAST CEILING DURING RAIN

Rain began at Portland 0015 EST, April 10, 1946. The lapse rates of temperature and wet-bulb temperature, as determined from the 2200 EST radiosonde of the 9th, are shown in figure 6. At 2300 EST, the pibal reached 9,000 feet, indicating winds of 150° to 210° at all levels. With these data, the forecast of ceiling is made as follows:

The temperature and depression lapse rates indicate that ceilings of (a) 9,000 feet, (b) 6,700 feet, and (c) 4,500 feet will occur if the rain is of sufficient duration. Ceilings of less than 4,500 feet are not indicated, therefore the other heights selected are (d) 800 feet, (e) 500 feet, and (f) 300 feet. The 800-foot ceiling height is selected because it has been found to be the most frequent low ceiling. The 500- and 300-foot ceilings are chosen for their significance to aviation.

Considering each ceiling in turn:

(a) *9,000 feet.*

From the depression lapse rate, $\tau'=0.9°$ F.

From the table in figure 4, $F_z=2.1°$ F. hr^{-1}.

Since $F_zt'=(2.1)\ (2.25)>\tau'$, equation (12) is applied: $t'_s=\tau'/F_z=0.4$ hrs. after the observation of τ', i. e., the 9000-foot ceiling will occur at *2224 EST.*

(b) *6,700 feet.*

From the depression lapse rate, $\tau'=3.6°$ F.

From the table in figure 4, $F_z=1.7°$ F. hr^{-1}.

$F_zt'=(1.7)\ (2.25)=3.8>\tau'$, so $t'_s=3.6/1.7=2.1$ hrs. after 2200 EST, or at *0006 EST.*

(c) *4,500 feet.*

$\tau'=3.2$, $F_z=1.4$; $F_zt'=3.2=\tau'$; therefore the 4,500-foot ceiling occurs at the time rain starts, i. e., at *0015 EST.*

(d) *800 feet.*

$\tau'=3.5$, $F_z=0.4$; $F_zt'=0.9<\tau'$; therefore applying equation (11): $\tau_0=\tau'-F_zt'=2.6$.

From figure 4, for $\tau_0=2.6$ and $F_z=0.4$, it is found that $t_s=3.5$ hrs., i. e., an 800-foot ceiling will occur 3.5 hrs. after the start of rain, or at *0345 EST.*

(e) *500 feet.*

$\tau'=2.8$, $F_z=0.16$, $\tau_0=2.4$, $t_s=5.7$ hrs. after rain starts, i. e., at *0600 EST.*

(f) *300 feet.*

$\tau'=2.4$, $F_z=0.08$, $\tau_0=2.2$, and $t_s=7.4$ hrs. after rain starts, or at *0745 EST.*

A comparison of this computed variation of ceiling with the actual ceiling variation is shown in figure 6.

ESTIMATE OF WET-BULB DEPRESSION IN LOWER LAYERS FROM SURFACE VALUES

Moisture gradients in the lower levels are usually variable in advance of rain, and since radiosonde data are relatively scarce, it is desirable to make use of surface data to predict the time of occurrence of low ceilings. It has been mentioned that the diurnal variation of the surface depression may be appreciable and should be considered. Surface values before the start of rain may not be a good measure of the value a few hundred feet from the ground, where the diurnal factors may be less important.

Generally, the range of the diurnal variation increases with the dryness of the air, so there should be very little diurnal variation during rain. However, near or before the start of rain, diurnal factors may cause a large variation. Thus, comparing the depression variation before the start of rain with the normal daily variation in figure 7, it is seen when rain begins in the early afternoon, the depression increases its value by about 75 percent from early morning to near noon; when rain begins near midnight, the depression lowers during the afternoon and evening at a rate not very different from the normal lowering.

A rough measure of the depression above the layer of diurnal influence may be made by assuming that the actual variation is proportional to the normal and applying a correction to the surface depression depending upon the time of day. These correction factors would, however, be unreliable when applied to depressions measured at night because there is very little correlation between the minimum surface depression reached at night and the upper air values. In order to relate surface depressions with upper air values, it is necessary to determine the intensity of the factors contributing to the diurnal variation. However, these factors, no matter how intense, have little or no influence on the surface depression at the time of day when the depression is normally at its average daily value. At these times of day, the surface depressions are best correlated with upper air values.

The times of day when the wet-bulb temperature depression has its average values may be determined readily by averaging 24 hourly surface depressions and finding the time of day this average occurs on the smoothed diurnal variation. Approximate average values obtained in this manner, but by using the difference between average hourly temperature and wet-bulb temperature values [6] for Washington, D. C., are shown in table 3.

TABLE 3.—*Time of day wet-bulb depression has its daily average value, Washington, D. C. (Eastern Standard Time)*

Month	A. M.	P. M.	Month	A. M.	P. M.	Month	A. M.	P. M.
Jan	1030	2130	May	0930	2030	Sep	0900	1930
Feb	1100	2230	June	0930	2030	Oct	0930	2000
Mar	1000	2230	July	0930	2030	Nov	1030	2030
Apr	0930	2130	Aug	0930	2030	Dec	1030	2030
						Annual	1000	2030

Thus, the depression is normally at its mean value at 1000 EST and again at 2030 EST. Since upper air observations are made at about 1030 and again at 2230 EST, it follows that the morning radiosondes are best suited for finding the correlation of upper air depressions and surface values. This is made obvious in figure 8, which contains scatter diagrams relating the wet-bulb depressions at the surface, 500 feet, and 1,000 feet for both the morning and evening observations taken within 12 hours before the start of rain. Further, since the 1030 EST points in both diagrams fall reasonably close to a straight line of unity slope, the vertical gradient of the wet-bulb temperature depression below 1,000 feet before rain starts, is nearly zero at the times of day when diurnally varying factors do not contribute to the wet-bulb depression.

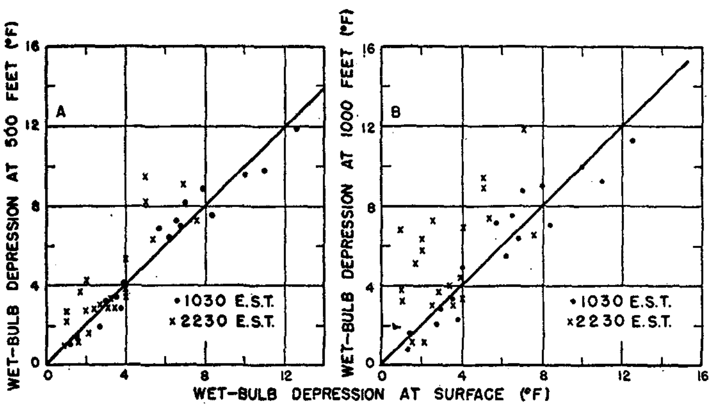

FIGURE 8.—Scatter diagrams for cases 0-12 hours before start of rain, relating wet-bulb temperature depression at (A) 500 feet and surface, and (B) 1,000 feet and surface. (Portland, Maine)

Thus, if rain starts soon after 1030, or 2030 EST, the surface value of the wet-bulb depression at these times may be assumed to give, approximately, the value of the depression up to 1,000 feet. This value may then be substituted for τ' in equation (11) to find the depression at any level below 1,000 feet at the time rain begins, if the appropriate value for F_z is used. It should be noted, however, that if rain begins long after 1030 EST (or 2030), the average values for F_z given in the table in figure 4 may be appreciably higher than the actual average, so that their use may lead to an under-estimate of τ_0 above the ground, in such cases.

FORECASTING TIME OF CEILING OCCURRENCE USING SURFACE WET-BULB DEPRESSION

To illustrate the application of surface elements measured before the start of rain to predict the time low ceilings will occur, the ceiling heights of (a) 800 feet, (b) 500 feet, and (c) 300 feet will be chosen for consideration. If the higher values for F_z are chosen from the table in figure 4, then equations (11) and (12), for each level become:

(a) *800 feet.* If $\tau'>0.4t'$, then

$$\tau_0=\tau'-0.40t' \tag{11a}$$

where τ' is the surface wet-bulb depression measured at 1030 EST, or 2030 EST, whichever is closest to the time rain starts, and t' is the number of hours to the start of rain. The number of hours, after the start of rain, when the 800-foot ceiling will first occur can then be found from figure 4 using the value of τ_0 given by equation (11a) and the value $F_z=0.40$.

If $\tau'\leq 0.4t'$, then the time the given low ceiling will first occur, in hours after the measurement of τ', is

$$t_s'=2.5\tau' \tag{12a}$$

Similarly for the other levels:

(b) *500 feet.* If $\tau'>0.16t'$, then

$$\tau_0=\tau'-0.16t' \tag{11b}$$

and if $\tau'\leq 0.16t'$, then

$$t'_s=6.3\tau' \tag{12b}$$

(3) *300 feet.* If $\tau'>0.08t'$, then

$$\tau_0=\tau'-0.08t' \tag{11c}$$

and if $\tau'\leq 0.08t'$, then

$$t'_s=12.5\tau' \tag{12c}$$

These formulas, together with figure 4, were tested on all cases of continuous rain which occurred at Boston during the years 1944 and 1945, assuming that the values of F_z found for Portland apply. Cases in which low ceilings occurred before the start of rain were omitted. The average of the wet-bulb depressions measured at 0930, 1030, and 1130 EST (or 1930, 2030, and 2130 EST) was taken for τ', in order to reduce possible errors in the measurement of the depression. For cases in which rain began close to 1030 or 2030 EST, the last two measurements before the start of rain were averaged so that the later data could be used. In any case, the average time of the averaged measurements to the time of beginning of rain was used for t'. The resulting forecasts are compared

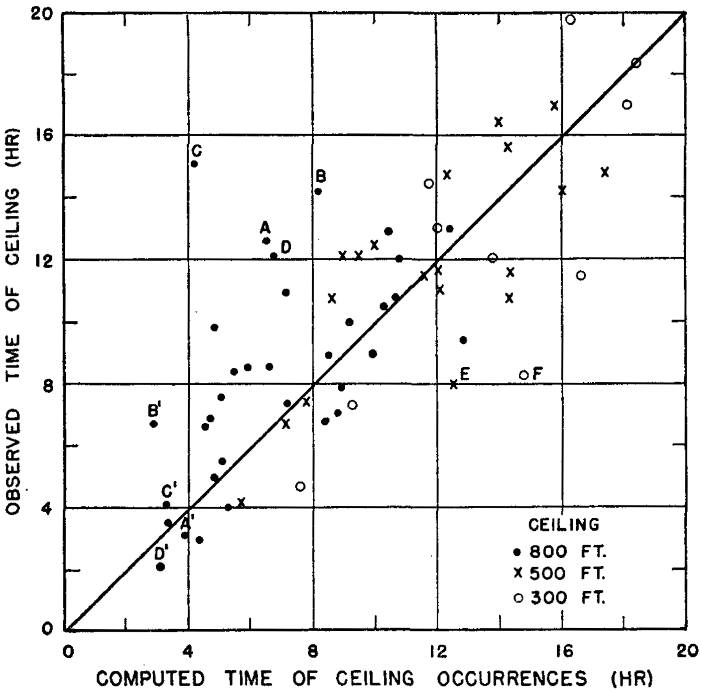

Figure 9.—Comparison of forecast with actual time of ceiling in rain at Boston, Mass., 1944 and 1945.

with the time that the ceiling was first reported, at or below the given height, in figure 9. In most cases the error is within 3 hours.

Points A, B, C, and D in figure 9 represent the only 800-foot cases for which t' is more than 8 hours. For all other cases t' is 8 hours or less. The large deviations in cases A, B, C, and D may be attributed to the error resulting from the application of the average F_z value over too great a time interval. If the surface values before the start of rain are used to give a better measure for the wet-bulb depression at 800 feet at the start of rain, the points A, B, C, D translate to A′, B′, C′, D′, respectively.

The two points E and F show the largest departure in the opposite direction. These correspond to a single case. Actually the ceiling lowered from 900 feet to 200 feet within 2 hours just ahead of a warm front, preceded by light winds and drizzle. It is to be expected that when a front lies in the vicinity of the station, before the time rain can produce low cloudiness, the properties of the front, with respect to low clouds and fog, determine the ceiling, and therefore should be considered in an actual forecast.

In many cases the ceiling did not lower to the selected levels, so not all the forecasts made appear in figure 9. To show how the formulas apply to all cases, the contingency table 4 was prepared. This table compares the lowest ceiling observed in each case with the minimum ceiling that would have been forecast if the time of ending of ceiling lowering was known; for example, the time dry air advection begins or the time rain ends. Since the computed minima are within a few hundred feet of the observed lowest ceilings, in most cases, it appears that conditions favoring saturation did not continue a sufficient time for the cloud to develop at the selected level.

Table 4.—*Computed and observed minimum ceiling in rain at Boston, Mass., 1944 and 1945*

		Number of cases forecast				
		Less than 400 feet	400 to 500 feet	600 to 800 feet	Over 800 feet	Total
Number cases observed	Less than 400 feet	7	2	1	0	10
	400 to 500 feet	3	5	1	0	9
	600 to 800 feet	1	4	5	1	11
	Over 800 feet	0	0	0	2	2
	Total	11	11	7	3	32

CONCLUSION

It is evident from this study that it is possible to predict the rate of ceiling lowering during rain, objectively and with a reasonable degree of accuracy. Although several important factors influencing cloud formation have either been neglected or roughly approximated in the derivation of the forecast method, a logical approach was attempted in order that the forecaster may at least make an approximate allowance for the effect of these factors, when known or extrapolated. The following list, although incomplete, suggests how forecasts made from figure 4 may be improved qualitatively by the forecaster and, of course, suggests the lines along which further study should be made in order to improve the accuracy of ceiling forecasts:

(1) If a front or trough, with which low ceilings are associated, lies nearby, then the ceiling may lower more rapidly than forecast.

(2) In cases of heavy rain or snow, near the beginning of precipitation, the ceiling will lower more rapidly than forecast.

(3) With strong turbulence near the ground the time of formation of very low clouds may be underestimated, while the time of formation of clouds near the top of the mixed layer may be overestimated.

(4) Ceilings may not lower as rapidly as forecast in cases of intermittent or showery precipitation.

(5) The time a ceiling of given height will occur may be underestimated if there is only slight advection of moist air at that level; if there is dry air advection, the ceiling may rise (e. g., near the time rain ends).

Perhaps the greatest improvement in forecasting the time of occurrence of the very low ceilings may be attained by (a) using F_z values indicated by wind and wet-bulb depression distributions, and (b) a study of frontal characteristics.

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REFERENCES

1. W. Findeisen, "Die Entstehung der 0°-Isothermie und die Fraktocumulus-Bildung unter Nimbostratus," *Meteorologische Zeitschrift*, Band 57, Heft 2, Februar 1940, s. 49–54. Translation by F. A. Berson, "The Formation of the 0°-isothermal Layer and Fractocumuli Under Nimbostratus," Meteorological Offices, London. Copy available in U. S. Weather Bureau Library.
2. S. Petterssen, *Weather Analysis and Forecasting*, McGraw-Hill Book Co., New York, 1940.
3. *Chemical Engineers Handbook*, McGraw-Hill Book Co., New York, 1934, p. 858.
4. R. Gunn and G. D. Kinzer, "The Terminal Velocity of Fall for Water Droplets in Stagnant Air," *Journal of Meteorology*, vol. 6, No. 4, August 1949, pp. 243–248.
5. P. Lenard, "Über Regen," *Meteorologische Zeitschrift*, Band 21, Heft 6, Juni 1905, s. 249–262.
6. U. S. Weather Bureau, "The Climatic Handbook for Washington, D. C.," *Technical Paper No. 8*, January 1, 1949.
7. Louis Goldman and Conrad P. Mook, "Forecasting Ceilings During Continuous Prewarm Front Rain," paper presented at 98th National Meeting of the American Meteorological Society, Washington, D. C., April 20–22, 1948.